Is the Area of a Triangle Dependent on Shape of the Triangle

Lesson Summary:
Have you ever had a student who stated emphatically that the area of a triangle changed with the shape, even though the base and height of the triangle did not change? This lesson, hopefully, will convince even the most skeptical that the area of a triangle is dependent only upon the length of the base and height of the triangle.

Key Words:
Area, triangle

Background knowledge:
familiarity with Cabri program; knowledge of the formula for the area of a triangle.

NCTM Standards addressed:
Analyze characteristics and properties of 2 dimensional geometric objects; select and use different representational systems, including coordinate geometry and graph theory; recognize the usefulness of transformations and symmetry in analyzing mathematical situations; use visualization and spatial reasoning to solve problems both within and outside mathematics.

Learning objectives:
The students will construct the requisite triangle, following the instructions; and discover that the area of a triangle is independent of its shape providing the length of the base and height remain constant.

Materials needed:
Cabri program

Suggested procedure:
a. Ask how many students believe that the area of a triangle is dependent upon the shape or type of triangle.
b. Students may be grouped as desired by the teacher, hopefully matching the less proficient with the more proficient students.
c. Assessment is left to the discretion of the instructor. Please keep in mind that this is learning by discovery, and frustration may be at peak levels before the real learning is accomplished. It is suggested that each student be assessed based upon how far he or she has come in relation to him/herself.
Is the Area of a Triangle Dependent on the Shape of the Triangle?

Team Members’ Names: ______________________________________________________________

File name: ________________________________________________________________________

**Goal:** to explore the relationship between the area of the triangle with a fixed base and altitude, and the movement of one vertex (A) along a line \( \ell \).

*Remember: Area of a triangle = \( \text{base} \times \text{height}/2 \)

**Procedures:**

1. Draw a line \( \ell \) (use the line tool).
2. Place one point on the line \( \ell \). Label the point A. (use point tool).
3. Place one point B below the line \( \ell \) (use the point tool).
4. Draw a line through point B and parallel to line \( \ell \) (use parallel line tool).
5. Place one point C on the line (use line tool).

\[ \text{A} \]
\[ \text{B} \]
\[ \text{C} \]

6. Draw segments \( \overline{AB} \), \( \overline{AC} \), and \( \overline{BC} \) (use segment tool).

\[ \text{A} \]
\[ \text{B} \]
\[ \text{C} \]

7. Draw a perpendicular line through A to line \( \overline{BC} \) (use perpendicular line tool).

*NOTE: Make sure the line is perpendicular to the line \( \overline{BC} \), not segment \( \overline{BC} \).*

8. Label the intersecting point D (use intersecting point tool).
9. Draw altitude \( \overline{AD} \) and make it a dotted segment (use segment tool).
10. Hide \( \overline{AD} \) (use Hide/Show tool).
11. Mark $\angle BDA$ (use mark angle tool).

12. Measure altitude $\overline{AD}$ and base $\overline{BC}$ (use measure tool).

13. Use calculator to find the area of the triangle.

14. Label the result from step 13 as follows: $(AB \times BC)/2 = \text{number shown}$ (use comments tool).

15. Grab point $A$, moving it along line $\ell$ (use pointer/arrow tool). Can you find a point where the base and altitude change?________________________________________________

16. Grab point $A$ so that the altitude $\overline{AD}$ is outside the triangle (use the pointer tool).

What kind of triangle is this?________________________________________________

What do you notice about the area of the triangle?________________________________________________

What do you notice about the altitude $\overline{AD}$ and base $\overline{BC}$?________________________________________________
17. Grab point A, moving it so altitude $AD$ is inside the triangle (use the pointer tool).

What kind of triangle is this?________________________________________________

What do you notice about the area of the triangle?_______________________________

What do you notice about the altitude $AD$ and base $BC$?_______________

18. Did the area of the triangle change with the change in shape of the triangle?_________
Why?_____________________________________________________________________
_____________________________________________________________________

19. Does the shape of the triangle affect the area of the triangle?______ Why or why not?
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________