Exploring the Hinge Theorem

Lesson Summary:
This guided activity allows students to discover the Hinge Theorem. The Hinge Theorem can be used to write inequalities between two triangles given two pairs of congruent sides. The Hinge Theorem states If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first triangle is greater than the included angle of the second triangle, then the third side of the first triangle is greater than the third side of the second triangle.

Key Words:
Hinge theorem, inequality, triangle

Background Knowledge:
Students must be familiar with Cabri geometry, either on the P.C> or on the TI-92 calculator. Students must also be able to identify corresponding congruent parts of triangles and understand basic triangle inequalities.

Ohio State Model Curriculum Objectives
1) Students will be able to compare, order, and determine equivalence of real numbers.
2) Students will be able to write inequalities for various triangular relationships.

Learning Objectives
1) Students will be able to use inequalities involving triangle side lengths and angle measures to solve problems.

Materials
Computers or calculators with Cabri geometry installed
Exploring the Hinge Theorem lab worksheet

Procedures
Review the various inequalities that can be written to describe relationships in triangles.
Divide the students into groups of no more than three (two is preferable)
Have students complete the lab worksheet.
Monitor students’ progress during the activity.
Use results from the lab activity as assessment.
Review findings prior to the conclusion of class.
Exploring the Hinge Theorem

Lab Worksheet

Team members: _______________________________________________________
File Name:  ___________________________________________________________

Lab Goals: Students will investigate and discover the Hinge Theorem, which describes inequalities for two triangles. The primary activity examines a special case of the Hinge Theorem and the extension looks at all possible cases of the Hinge Theorem.

Procedures

1) Draw a circle C.

2) Place points A and B on circle C.

3) Draw segments $\overline{AC}$, $\overline{BC}$, and $\overline{AB}$ to form triangle $\triangle ABC$.

4) Grab point A or point B and move it along circle C. Watch what happens to the length of segment $\overline{AB}$. Describe what happens to segment $\overline{AB}$ as the measure of angle $\angle C$ changes.
5) Draw another triangle \( \triangle DCE \) following the same steps above.

6) List the congruent corresponding parts of the two triangles. _______________

__________________________________________________________________

7) Grab the points and move them such that angle \( \angle ACB \) is larger than \( \angle DCE \). How do segments \( \overline{AB} \) and \( \overline{DE} \) compare? _______________________________

Now grab the points and move them such that angle \( \angle ACB \) is smaller than angle \( \angle DCE \). Now how do segments \( \overline{AB} \) and \( \overline{DE} \) compare? __________________

8) The first activity had you examine a special case of the Hinge Theorem where the two triangles were isosceles and had congruent sides. Explain why you know that the triangles are isosceles with congruent sides.

__________________________________________________________________

__________________________________________________________________

9) Finish the Hinge Theorem: If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first triangle is greater than the included angle of the second triangle, then
**Extension: Exploring other cases of the Hinge Theorem**

As an extension, create two non-isosceles triangles using Cabri. Design triangle $\triangle ABC$ and triangle $\triangle DCE$ so that segments $AB$ and $AD$ are congruent and segments $AC$ and $AE$ are congruent. Follow the procedures below.

1) Draw two concentric circles. Concentric circles have the same center and difference radii.

![Diagram of two concentric circles with points A, B, and C labeled on the smaller circle, and points A, D, and E labeled on the larger circle.](use circle tool)

2) Draw segments $AB$ with one endpoint at A and the second endpoint B on the smaller circle. Then draw segment $AC$ with one endpoint at A and the second endpoint C on the larger circle. Next, draw segment $BC$ to form triangle $\triangle ABC$.

![Diagram showing triangle ABC with segments AB, AC, and BC drawn.](use segment tool)

3) Repeat step 2 to form a second triangle $\triangle ADE$. Make segment $AB$ congruent to segment $AD$ congruent and segment $AC$ congruent to segment $AE$.

![Diagram showing triangle ADE with segments AB, AD, and AE drawn.](use segment tool)

Explain why this construction guarantees that segments $AB$ and $AD$ are congruent and segments $AE$ and $AC$ are congruent.
4) Now measure angle $\angle BAC$ and angle $\angle DAE$. Next find the length of segment $BC$ and segment $DE$. Label the values appropriately. Drag the values to the side.

5) Make angle $\angle BAC$ larger than angle $\angle DAE$. How does the length of segment $DE$ compare to the length of segment $BC$?

______________________________________________________________________________
______________________________________________________________________________

6) Make a conjecture about the length of segment $DE$ compared to segment $BC$ if angle $\angle DAE$ is smaller than angle $\angle BAC$.

______________________________________________________________________________
Try this. Is your conjecture accurate? ______________

7) In your own words, write the Hinge Theorem. ______________________________
______________________________________________________________________________
______________________________________________________________________________

8) Why is it called the Hinge Theorem? ______________________________
______________________________________________________________________________