Secant Line Approximations of the Tangent Line

**Introduction**
This lesson is intended as an introduction to the derivative as the slope of a tangent line. It leads the student to recognize the tangent line at a point P on a curve is the limit of the secant lines that pass through P and another point on the curve Q, as Q approaches P.

**Key Words:**
Tangent, calculus, derivative

**NCTM Strands**
*Use visualization, spatial reasoning, and geometric modeling to solve problems.*
*Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.*
*Apply appropriate techniques, tools, and formulas to determine measurements.*
*Analyze change in various contexts.*

**Learning Objective**
The goal of this lab is for students to recognize that the slope of a tangent line at a point P on a given curve is the limit of the slopes of the secant lines that pass through P and a second point Q, as Q approaches P.

**Materials**
Computers with Cabri Geometry.

**Procedures**
**Problem:** The idea of instantaneous rate of change is often discussed in the Calculus classroom, but it is not often demonstrated sufficiently. This lesson can help visual students to see the limiting relationship between the secant lines and tangent lines. A good introduction would be to discuss a problem of average velocities, and how they can lead to the calculation of instantaneous velocity.

**Grouping:** Students should be grouped in pairs.

**Assessment:** Assessment is left up to the individual teacher, but should include some reasonable form of authentic assessment, which consists of demonstration of the concept learned, as well as a verbal description of the processes developed.
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**Goals:** The goal of this lab is for students to recognize that the slope of a tangent line at a point P on a given curve is the limit of the slopes of the secant lines that pass through P and a second point Q, as Q approaches P.

**Investigation:**

1. Open Cabri, show axes, and create an arc as in the figure below.

2. Construct points P and Q on the arc.

3. Draw a line parallel to the x-axis through P and a line perpendicular to the x-axis through Q as shown. Place a point at the intersection and label it D.

4. Hide both lines and construct two segments PD and QD

5. Draw a secant line to the arc that passes through P and Q.

6. Measure the length of both segments, labeling PD as $dx$ and QD as $dy$. (This is common notation, letting $dx$ represent the change in $x$ and $dy$ represent the change in $y$)

7. Calculate the slope of your secant line with a calculation of $dy/dx$. (Change in $y$ over change in $x$)
8. Let \( Q \) approach \( P \) from the right until \( dx \) measures the values in the table at right (as close as possible) and record the corresponding slope values (\( dy/dx \)).

<table>
<thead>
<tr>
<th>( dx )</th>
<th>( dy/dx )</th>
</tr>
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<tbody>
<tr>
<td>3</td>
<td></td>
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<tr>
<td>2</td>
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<td>1</td>
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<td>.5</td>
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9. If \( dx \) equals zero, the secant line becomes a tangent line. Based on your table, what would you approximate the slope of the tangent line to be at \( P \)?

10. Would a slope calculation be possible if \( dx \) actually equaled zero? Why?

11. Change the location of \( P \), and repeat the process above to approximate the slope of the tangent line at the new \( P \) value.

12. Is this slope different than the slope at your first \( P \) value? Should it be? Why?

13. Describe how you could approximate the slope of a tangent line to a curve at a point \( P \) just by looking at the curve and the location.

14. What limitation does Cabri have that is illustrated by the following drawing? Specifically, how is the slope incorrect for this situation?

\[ y = x^2 \]
\[ a = .08 \text{ cm} \]
\[ d = 1.24 \text{ cm} \]
\[ dy/dx = 0.27 \]

**Extensions:**

1. Describe how you would use this process if you were given an equation and asked to calculate the slope of the tangent line at a given x-value.

2. Calculate the slope of the tangent line to \( y = x^2 \) at \( x = 1, 2, 3 \), and 4. How do these slopes compare to the x-values where they are calculated.