Zeros (Roots) of Polynomials

1. Using your calculator, draw the graphs for the following functions. At what point(s) do the graphs cross the x-axis?

   a) \( y = 2x + 4 \)  
   b) \( y = 4x - 3 \)  
   c) \( y = x^2 - 4 \)  
   d) \( y = 2x^2 + 3x - 6 \)  
   e) \( y = 3x^3 - 5x^2 - 26x - 8 \)  
   f) \( y = 4x^3 - 12x^2 + 3x + 5 \)

2. What do the coordinates of these points have in common?

   The x-coordinates of the points where the graph crosses the x-axis are called the zeroes or real roots of the function.

3. For each of the functions from #1, factor the right hand side of the expression (e and f are done for you!) and solve the equations for \( y = 0 \).

   a) \( y = 2x + 4 \)  
   b) \( y = 4x - 3 \)  
   c) \( y = x^2 - 4 \)  
   d) \( y = 2x^2 + 3x - 6 \)  
   e) \( y = 3x^3 - 5x^2 - 26x - 8 \)  
   f) \( y = 4x^3 - 12x^2 + 3x + 5 \)

4. What do you notice about the solutions in #3 and the points you found in #1?
5. If the polynomials are factored, how can you guess the zeroes without graphing?

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6. Look at your answers in #1. How many times did each graph cross the x-axis? Do you see any relationship between the degree of the polynomial function and the number of times the graph crosses the x-axis?

   a) ____________________  b) ____________________
   c) ____________________  d) ____________________
   e) ____________________  f) ____________________

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7. Write an equation for a polynomial function which has zeroes of -2 and 5. Think about what degree this polynomial will be. Enter your equation into your calculator to check your answer graphically.

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8. Write a polynomial function of least degree which has zeroes at -1, 2, and 5. What degree will this polynomial be? Enter your equation into your calculator to check your answer graphically.

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9. Graph the functions \( y = (x - 3)(x + 4), \)
   \[ y = 3(x - 3)(x + 4), \] and
   \[ y = 0.5(x - 3)(x + 4). \]
   Do they have different zeroes? What can you conclude about other functions represented by \( y = k(x - 3)(x + 4), \) where the constant \( k \) represents any real number.
10. Write two more possible equations for the polynomial with same zeroes described in #7, one with the same degree and another one of different degree.

11. Write two more possible equations for the polynomial with the same zeroes described in #8, one with the same degree and another one of different degree.

12. Write an expression for the function represented by each of the following graphs. Enter your equation into your calculator to check your answer graphically.

A. 
B. 

13. How many zeroes do the following equations have? Graph each equation to check your answer.

   \[ y = 12x^2 - 6x - 6 \]

   \[ y = 6x^4 + 5x^3 - 15x^2 + 4 \]

   \[ y = 2x^5 + x^4 - 39x^3 - 6x^2 + 80x - 32 \]

   \[ y = 6x^3 + 11x^2 - 4x - 4 \]
14. Consider the function $y = x^2 - 4x + 4$. How many zeroes do you think this function could have? Now graph the function. Does it cross the $x$-axis as many times as you thought it would? Why do you think this happened?

When you factor the expression, you get $y = (x - 2)(x - 2) = (x - 2)^2$. There are still two real roots (2 and 2) but they are not distinct numbers. This happens any time the factors are the same. We call $x = 2$ a **double root**.

15. The degree of a polynomial tells you the **most number of times** the graph of a function will cross the $x$-axis. There could be fewer real roots, but never more. How many times would you expect the graph of $y = 2x^2 + 2x + 4$ to cross the $x$-axis? How many times does it cross? How many real roots does this function have?

16. Consider the function $y = x^3 + 4x^2 + 3x + 5$. How many roots do you think this function could have? How many times does the graph actually cross the $x$-axis? How many real roots does this function have?