

# Math 3450:636

## Homework 1

1) Using the Havel-Hakimi algorithm, determine if  $\pi = (6, 5, 5, 4, 3, 3, 2)$  is graphic. If it is, construct a realization in the manner described in class. Verify that the sequence is or is not graphic using the Erdős-Gallai criteria.

2) Chartrand and Lesniak, p11, **1.2**

3) Let  $x \geq 1$  be an integer, and let  $\pi = (d_1, \dots, d_n)$  be a nonincreasing sequence where

$$d_1 = d_2 = \dots = d_i = x$$

for some  $1 \leq i \leq n$  and

$$d_{i+1} = \dots = d_n = (x - 1)$$

. Assume also that  $x \leq n - 1$  and that the sum of the  $d_i$  is even. Prove that  $\pi$  is a graphic sequence. (Hint: Use induction and the Havel-Hakimi algorithm).

4) Write the Havel-Hakimi algorithm in either code or pseudocode (pseudocode is my preference) and give an asymptotic estimate of its running time.

5) (A variation on Chartrand and Lesniak, p65, **3.3**) Show that a graph  $G$  is a forest if, and only if, every connected subgraph is an induced subgraph.